Contract design for a supply chain with the potential scarcity of components

Yungyi Li
Institute of Industrial Engineering
National Taiwan University
1, Section 4, Roosevelt Road
Taipei, 106, Taiwan

I-Hsuan Hong*
Institute of Industrial Engineering
National Taiwan University
1, Section 4, Roosevelt Road
Taipei, 106, Taiwan

Sunny S. Yang
Department of Business Administration, College of Management
National Taiwan University
1, Section 4, Roosevelt Road
Taipei, 106, Taiwan

Ting-Ju Ou
Institute of Industrial Engineering
National Taiwan University
1, Section 4, Roosevelt Road
Taipei, 106, Taiwan

* Corresponding author: ihong@ntu.edu.tw, phone: +886-2-3366-9507, fax: +886-2-2362-5856

The corresponding author holds a joint appointment with the Department of Mechanical Engineering, National Taiwan University.

Contract design for a supply chain with the potential scarcity of components

Abstract

Contracts pertaining to current business markets are highly varied. Much work has been done on contracts with regard to the study of the negative relationship between price and quantity, but little has been done that considers the positive and negative relationship together. This article investigates contracts for potentially scarce components with two charging schemes. A scarce component occurs in supply or demand disruption, with the price and total ordering quantity positively correlated. Other components we call non-scarce components, where the price and total ordering quantity are negatively correlated. A potentially scarce component means that a component may be scarce in the future but no one can predict when. This research applies the concept of the Stackelberg-type model with or without Cournot competition in the construction of a two-tier supply chain model. The upstream supplier acting as the leader decides upon a contract for potentially scarce components. Then the downstream manufacturers, being informed of contract information and market situations, place an order for the components. We analytically show that our contract can be applied to potentially scarce components with the probability of supply disruption. The strategies for the supplier are provided to compare profits and decisions based on concerns over the supply chain disruption. Some characteristics of the equilibrium decisions and managerial insights are suggested for the supplier.

Keywords: Supply chain management, Stackelberg model, positive correlation between price and quantity, scarce component, supply chain disruption

1. Introduction

Optimal supply chain performance has been extensively studied for some time. A supply chain is a system of organizations, people, activities, information and resources, set up to improve production. Determining how to coordinate every independent economic entity for optimal system performance is the critical issue in supply chain management (Li and Wang, 2007; El Ouardighi and Erickson, 2015). Lau e al. (2007) and Hou et al. (2010) discuss supply chain coordination of return (buy-back) contracts, with a risk-sharing mechanism for retailers returning to manufacturers all or some of unsold products to get credits. Sales rebate contracts are discussed in Taylor (2002) and Wong et al. (2009), where manufacturers pay a bonus to the retailer for each unit sold. Taylor and Xiao (2009) compare rebate contracts with returns contracts in the context of the retailer forecasting with a forecasting cost to improve customer demand information. A revenue-sharing contract is a retailer paying a supplier a wholesale price for each unit and where some revenue is gained by the retailer (Cachon and Lariviere, 2005; Wang et al., 2004). Some literatures (Lian and Deshmukh, 2009; Ghadge et al., 2017) focus on quantity-flexibility (QF or options) contracts. These are concerned with purchasing excess units and setting an option reservation quantity to resolve demand uncertainty. The firm then flexibly exercises some fraction of the reserved option. Under a quantity discount contract, the supplier offers a discount price to induce the retailer to order the system optimal quantity. The retailer has to trade-off the price reduction against the increase of the cost caused by inventories (Chen, 2015; Won, 2017).

As supply chains become more complex and expand overseas, they become much more vulnerable. Chopra and Sodhi (2004) categorize a variety of risks in supply chains, including disruptions, systems, procurement, inventory, etc. In addition, recycling operations also are complex processes; De Giovanni (2018) proposed a joint maximization incentive allowing the closed-loop supply chain to achieve a triple bottom line so that retailers are economically better off through its implementation. Tomlin (2009) shows that to a certain extent a product’s lifecycle and the length of disruption are two key factors for supply disruption. Taking a short lifecycle product (toy industry) as an example, disruption coinciding with the selling season could well have disastrous effects on Christmas products. Hendricks and Singhal (2005) indicate negative effects on shareholder value and on operating performance due to supply chain disruptions and comment that recovering from disruptions takes a long time and constant effort. Srinivasan, Mukherjee and Gaur (2011) state that a good partnership quality enhances supply chain performance and they analyze how supply risk, demand risk and environmental uncertainty moderate the relationship between partnership quality and supply chain performance.

Moreover, various literatures have contributed to information about how to manage supply chain disruption. Some research has discussed supply chain problem on demand disruption (Salema et al, 2007; Chen and Xiao, 2009). Regarding the problem of supply uncertainty, Tomlin (2006), for example, studies three supply-side tactics, namely sourcing mitigation, inventory mitigation and rerouting mitigation, to mitigate risk and to determine a firm's optimal disruption-management strategy. Sourcing from either an unreliable supplier or a reliable but more expensive supplier is so-called sourcing mitigation. Ang et al. (2016) focus on the sourcing strategy of a manufacturer managing disruption risk in a multi-tier supply chain with Tier 1 suppliers and Tier 2 suppliers. A rerouting mitigation is to shift either production or transportation methods after a disruption occurs.

Utility bills, typically, exhibit a positive relationship between unit price and the quantity consumed (positive price-quantity relationship), where the greater the usage of utilities, the higher is the price per unit. Several studies have discussed such positive relationships of water or electricity household consumption (Boland and Whittington 2000; Olson et al. 2003; Banal-Estañol and Micola 2011). Similar phenomena can be found in the case of luxury goods (McClure and Kumcu 2008). However, the research papers we reviewed above, relating to terms of supply chain coordination and supply chain disruption, have been discussed under the assumption of a decrease in the unit price with the quantity demanded. Only a limited amount of literature has discussed the issue of contract design for consuming products relative to the positive and negative price-quantity relationship under the probability of the supply chain disruption situation. A shortage of production capacity results in fierce competition for scarce components and long lasting ripple effects across the marketplace.

To address this research gap, this study aims to examine and to optimize the layout of a contract for an upstream supplier selling potentially scarce components to downstream manufacturers where the supplier cannot predict when the components are likely to become scarce (see Fig. 1). Alonso et al. (2007) investigate the causes of material shortage and when limitations are expected to emerge. Our study applies the knowledge of game theory and the nonlinear form to develop a contract model where the price rises with the quantity demanded as a supply chain disruption occurs; otherwise, we apply the rule of quantity discount for great demand. Specifically, based on industry experiences, we seek to study the quantity relationship of manufacturers' decisions rather than manufacturers deciding the price. By doing so, we intend to assist a supplier's decision for adopting optimal strategy when facing a supply chain disruption that may occur in the future and to develop an informed relationship with downstream manufacturers.

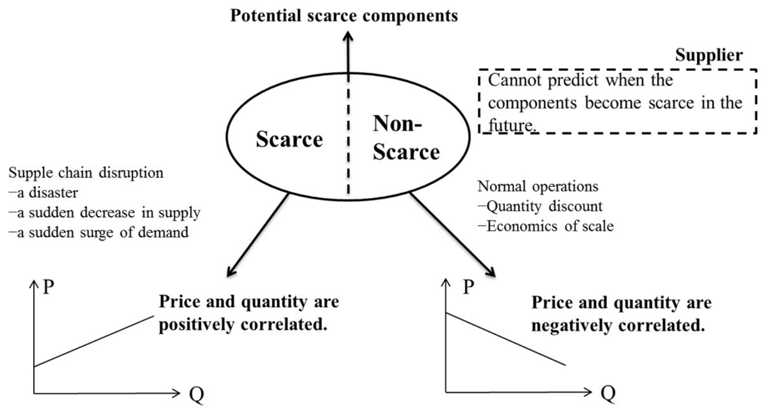

Fig. 1. Problem description

2. Model description and notations

This research applies the concept of the Stackelberg-type model with or without Cournot competition to construct a two-tier supply chain model. To specify supply chain structure we make the following assumptions allowing us to simplify the mathematics and yet capture the essence of the problem. The supply chain consists of an upstream supplier selling homogeneous potentially scarce components to a number of downstream manufacturers. A scarce component is a component with a very limited capacity constraint when an unexpected disruption occurs in a demand or supply side. Other components we call non-scarce components. A potentially scarce component is defined as a component that may become scarce in the future, and where the timing of the scarcity cannot be predicted. The supplier decides the price-quantity contract. Each manufacturer, under mechanisms of centralization and decentralization, engages in an uncooperative game against other manufacturers. It is a one-shot game. In other words, there is only one transaction in each period and the manufacturers are not allowed to order twice in a short-time period to prevent from arbitrage (manufacturers purchase a small quantity of the components at a low price on several occasions).

Let P_b be the price in the business to business (B2B)upstream market as shown in (1). We denote φ as a real probability of supply chain disruption. In terms of a scarce component, the price and total ordering quantity are positively correlated. The standard price (the base level of the price, i.e. the price where there is no quantity demanded) is a_1 where $a_1 > 0$ and the sensitivity of the price with respect to the quantity demanded (contract variable in our following content) is b where $b > 0$. In terms of a non-scarce component, the price and total ordering quantity are negatively correlated. We let the standard price be a_2 where $a_2 > 0$ and normalize the sensitivity of the price with respect to the quantity demanded to be -1. Let q_i be the ordering quantity for manufacturer i and $Q = \sum_{i=1}^{n} q_i$ be the total ordering quantity from *n* manufacturers.

$$P_b = \begin{cases} a_1 + bQ, & w.p. \quad \varphi \\ a_2 - Q, & w.p. \quad 1-\varphi \end{cases} \tag{1}$$

This study in terms of the upstream charging scheme is an extension of Hong et al.(2018), where the supplier charges for the components based on two charging schemes, namely the uniform charging scheme and the block charging scheme (see Fig. 2). On the one hand, under the uniform charging scheme, the supplier, based on the total ordering quantity (Q) of the potentially scarce component, sells at a single price (P_b) to the manufacturers. The total revenue of the supplier is the shaded area in Fig. 2. On the other hand, the manufacturers under the block charging scheme pay the varying P_b with the different Q.

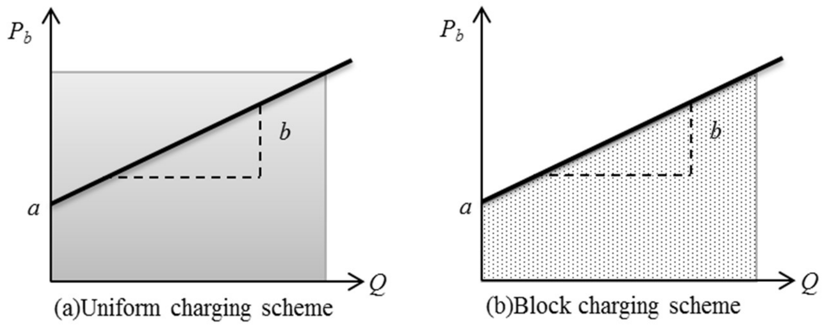

Fig. 2. The difference between uniform and block charging schemes

The expected market value of the upstream market is denoted as $\alpha = \varphi a_1 + (1-\varphi)a_2$, indicating the potential market size. For the supplier's production costs, we denote c_s as the production cost of the scarce components and denote c_{ns} as that of the non-scarce components. The expected production cost of the components for the supplier is $\varphi c_s + (1-\varphi)c_{ns}$, anddenote $\beta = \varphi c_s + (1-\varphi)c_{ns} + 2s$. And for manufacturer i, denotes the non-negative value c_i as its production cost.

Although a supply chain disruption caused by an acute shortage of key components considerably affects upstream transactions between supplier and manufacturers, downstream transactions between manufacturers and customers would not be influenced by the supply chain disruption. Let P_m be market price of the products characterized by a linear function,

$P_m = r - sQ$. Following the law of demand in the normal-product case, the demand curve is generally downward-sloping; in other words, as price decreases, consumers are willing to buy more products. There is a negative correlation between the choke-off price (the base level of the market price, i.e. the price at which there is no demand, denoted by r) and the sensitivity of the price with respect to the quantity demanded (s) where $r > 0$ and $s > 0$. Similarly, the base level of the market price is r, indicating the potential market size of the final market.

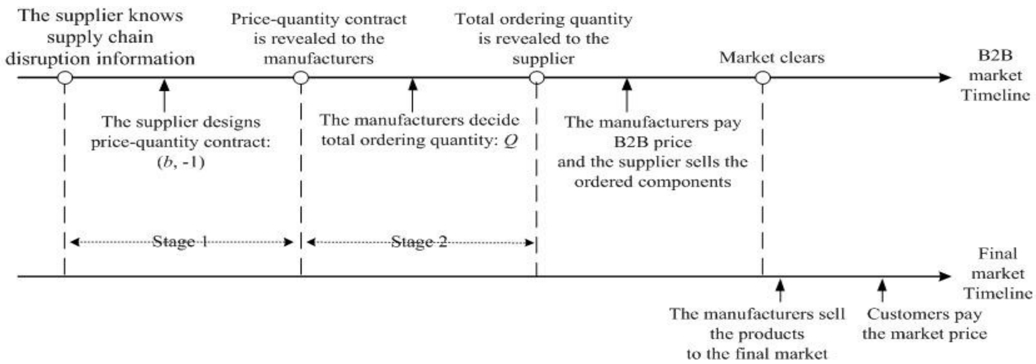

Fig. 3. The decision timeline for the supplier and manufacturers

Fig. 3 shows the decision timeline for the supplier and manufacturers during the period from the supplier offering the contract for the manufacturers to the final products selling to customers. The supplier is the leader and the manufacturers are the followers. The supplier offers a contract and the manufacturers accept or reject the contract. Guaranteeing the stability of the key component supply, the supplier has great bargaining power to determine the forms of contracts. Specifically, as an excess of supply occurs, the price and total ordering quantity are negatively correlated while they are positively correlated when suffering from a shortage of components.

The following sequence of events occurs in this game. The supplier, at the first stage, considers the probability of the supply chain disruption and chooses the contract variable (b) as the price-quantity relationship of the contract form when the positive correlation occurs and normalizes -1 as the relationship in the negative correlation situation. Assuming the manufacturers accept the contract, at the second stage, each manufacturer, being informed of contract information (contract variables of the price-quantity contract, b and -1) and market situations (the probability of supply chain disruption, φ), places an order for components (ordering quantity, q_i) to maximize its own profits. Next, the supplier delivers the ordered components to the manufacturers and charges the manufacturers the B2B price (P_b) for the components based upon the agreed contract. Finally, the manufacturers sell the final products to customers, with the market price (P_m) and total ordering quantity (Q). Nevertheless, if the manufacturers reject the contract, the game ends and both the supplier and manufacturers gain nothing. We summarize the notations in Table 1.

Table 1. Notation definitions

Notation	Definition
φ	The probability of supply chain disruption
P_b	The upstream B2B price between supplier and manufacturer
P_m	The downstream market price between manufacturer and consumer
a_1	The base level of the price (standard price) of the scarce component
a_2	The base level of the price (standard price) of the non-scarce component
b	The sensitivity of the price with respect to the quantity demanded (contract variable)
r	The base level of the market price (choke-off price)
s	The sensitivity of the price with respect to the quantity demanded by the consumer
c_i	The production cost for the i^{th} manufacturer
c_s	The production cost of the scarce components for the supplier
c_{ns}	The production cost of the non-scarce components for the supplier ($c_s > c_{ns} > 0$)
Δ	The increment of production cost in the small-scale manufacturer
Q	The total ordering quantity for all manufacturers
q_i	The ordering quantity for the i^{th} manufacturer

The remainder of this study is organized as follows. In Section 3 and 4, we develop two different manufacturer model and multiple identical manufacturer model under two charging schemes. ~~Then, we have numerical illustration in Section 5~~. Finally, we draw conclusions.

3. Two Different Manufacturers

In this section, we discuss the models that an upstream supplier sells homogeneous potentially scarce components to two distinctly differently sized manufacturers, where the large-scale manufacturer has a small unit production cost c, $c > 0$ and where the small-scale manufacturer has a large unit production cost $c + \Delta$, $c > 0$ and $\Delta > 0$. In terms of the ordering quantity for each manufacturer, this is denoted by q_1 and q_2 for large-scale and small-scale manufacturers respectively.

3.1 Centralized Manufacturers

Firstly we discuss the centralized manufacturer case with a central planner responsible for optimizing the manufacturers' overall performances, $\Pi_{d_1} + \Pi_{d_2}$. The total ordering quantity is $Q = q_1 + q_2$.

3.1.1 Uniform Charging Scheme

In the Stackelberg game, once the decision from the supplier is fixed, the manufacturers face a simple decision problem. Accordingly, we solve the manufacturers' optimal choice of any decision from the supplier (see Lemma 1) and then work backward to find the optimal choice for the supplier (see Corollary 1). We solve the two-stage sequential game by applying a backward induction, moving from the manufacturers' decisions at the second stage to the supplier's decision problem at the first stage.

With respect to the symmetric case, let $c = c_i$ be the production cost for each manufacturer *i*. The total production costs are composed of total fixed costs and total variable costs. The total fixed costs, like premises and machinery, are constant as output changes. The total variable costs increase at an accelerating rate as output increases because of the law of diminishing marginal returns, like the increased number of workers. Two of the most popular forms of the production cost are the quadratic and the transcendental logarithmic (Phillips, 2014). Consequently, the production cost function for the manufacturers used in this study is assumed to be quadratic, $\frac{cq_i^2}{2}$. The expected total profit for the centralized manufacturers is given by

$$\max_{q_1,q_2} \Pi_{d_1} + \Pi_{d_2} = \left\{ \left[P_m - \varphi\left(a_1 + bQ\right) - (1-\varphi)\left(a_2 - Q\right) \right] q_1 - \frac{cq_1^2}{2} \right\} + \left\{ \left[P_m - \varphi\left(a_1 + bQ\right) - (1-\varphi)\left(a_2 - Q\right) \right] q_2 - \frac{(c+\Delta)q_2^2}{2} \right\}, \quad (2)$$

where $P_m = r - sQ$ and $Q = q_1 + q_2$.

The above summation in the objective function is the expected revenue minus the expected cost of the component bought from the supplier and minus the production cost in the case of the two different business scales of the manufacturers.

Lemma 1. Given the contract variable b of the price-quantity contract, if $H(q_1, q_2) = \begin{pmatrix} -2b\varphi - 2\varphi - 2s + 2 - c, -2b\varphi - 2\varphi - 2s + 2 \\ -2b\varphi - 2\varphi - 2s + 2, -2b\varphi - 2\varphi - 2s + 2 - c - \Delta \end{pmatrix}$ is negative semi-definite, then the best responses of the large-scale and small-scale manufacturers under the uniform charging scheme are:

$$q_1^* = \frac{(r-\alpha)(c+\Delta)}{2\Delta b\varphi + 4bc\varphi + \Delta c + 2\Delta\varphi + 2\Delta s + c^2 + 4c\varphi + 4cs - 2\Delta - 4c} \quad (3)$$

$$q_2^* = \frac{c(r-\alpha)}{2\Delta b\varphi + 4bc\varphi + \Delta c + 2\Delta\varphi + 2\Delta s + c^2 + 4c\varphi + 4cs - 2\Delta - 4c} \quad (4)$$

where $\alpha = \varphi a_1 + (1-\varphi)a_2$.

We have the relationship of the ordering quantity for two manufacturers (q_1^* and q_2^*) at the second stage and then we return to the first stage to solve the optimal decision for the supplier. The supplier's expected profit function is given by

$$\max_{b} \Pi_{up} = \varphi\left[\left(a_1 + bQ\right)Q - \frac{c_s Q^2}{2}\right] + (1-\varphi)\left[\left(a_2 - Q\right)Q - \frac{c_{ns} Q^2}{2}\right], \quad (5)$$

where $Q = q_1 + q_2$.

The supplier proposes the contract with the different forms of contract variables of the price-quantity contract (b and -1) and anticipates that both of the manufacturers would respond accordingly to the decision from the supplier by maximizing the manufacturers' own profits (choose their own best responses, q_1^* and q_2^*).

Corollary 1. Under the uniform charging scheme, the results of the centralized two different manufacturer model are given below.

$$b^* = -\frac{1}{2}\frac{2(\Delta + 2c)\left[\alpha(-1+\beta+\varphi+s) - r(1+\beta-\varphi-s)\right] + c(\Delta+c)(3\alpha - r)}{\varphi(r+\alpha)(\Delta+2c)} \quad (6)$$

$$\left(q_1^*, q_2^*\right) = \left(\frac{1}{2}\frac{(\Delta + c)(r + \alpha)}{(c + \beta)(\Delta + c) + c\beta}, \frac{1}{2}\frac{c(r + \alpha)}{(c + \beta)(\Delta + c) + c\beta}\right) \tag{7}$$

$$\Pi_{up}^* = \frac{1}{8}\frac{\left(\Delta + 2c\right)^2 (\alpha + r)}{(c + \beta)(\Delta + c) + c\beta} \tag{8}$$

$$\left(\Pi_{d_1}^*, \Pi_{d_2}^*\right) = \left(\frac{1}{4}\frac{(\Delta + c)(r - \alpha)(r + \alpha)}{(c + \beta)(\Delta + c) + c\beta}, \frac{1}{4}\frac{c(r - \alpha)(r + \alpha)}{(c + \beta)(\Delta + c) + c\beta}\right) \tag{9}$$

3.1.2 Block Charging Scheme

Under the block charging scheme, the expected total profit for the centralized manufacturers is given by

$$\begin{aligned} \max_{q_1, q_2} \Pi_{d_1} + \Pi_{d_2} = &\left[P_m - \varphi\frac{\left(2a_1 + bQ\right)}{2} - \left(1 - \varphi\right)\frac{\left(2a_2 - Q\right)}{2}\right]q_1 - \frac{cq_1^2}{2} \\ &+ \left[P_m - \varphi\frac{\left(2a_1 + bQ\right)}{2} - \left(1 - \varphi\right)\frac{\left(2a_2 - Q\right)}{2}\right]q_2 - \frac{\left(c + \Delta\right)q_2^2}{2} \end{aligned}, \tag{10}$$

where $P_m = r - sQ$ and $Q = q_1 + q_2$.

The above summation in the objective function is the expected revenue minus the expected cost of the component bought from the supplier and minus the production cost in the case of the two different business scales of the manufacturers.

Lemma 2. Given the contract variable (b) of the price-quantity contract, if $H(q_1, q_2) = \begin{pmatrix} -b\varphi - \varphi - 2s + 1 - c, -b\varphi - \varphi - 2s + 1 \\ -b\varphi - \varphi - 2s + 1, -b\varphi - \varphi - 2s + 1 - c - \Delta \end{pmatrix}$ is negative semi-definite, then the best responses of the large-scale and small-scale manufacturers under the block charging scheme are:

$$q_1^* = \frac{(r - \alpha)(c + \Delta)}{\Delta b\varphi + 2bc\varphi + \Delta c + \Delta\varphi + 2\Delta s + c^2 + 2c\varphi + 4cs - \Delta - 2c} \tag{11}$$

$$q_2^* = \frac{c(r - \alpha)}{\Delta b\varphi + 2bc\varphi + \Delta c + \Delta\varphi + 2\Delta s + c^2 + 2c\varphi + 4cs - \Delta - 2c} \tag{12}$$

where $\alpha = \varphi a_1 + \left(1 - \varphi\right)a_2$.

We have the relationship of the ordering quantity for two manufacturers (q_1^* and q_2^*) at the second stage and then move back to the first stage to solve the optimal decision for the supplier. The supplier's expected profit function is given by

$$\max_b \Pi_{up} = \varphi\left[\frac{\left(2a_1 + bQ\right)}{2}Q - \frac{c_s Q^2}{2}\right] + \left(1 - \varphi\right)\left[\frac{\left(2a_2 - Q\right)}{2}Q - \frac{c_{ns}Q^2}{2}\right], \tag{13}$$

where $Q = q_1 + q_2$.

The supplier proposes the contract with the different forms of contract variables of the price-quantity contract (b and -1) and anticipates that both of the manufacturers would respond accordingly to the decision from the supplier by maximizing the manufacturers' own profits (choosing their own best responses, q_1^* and q_2^*).

Corollary 2. Under the block charging scheme, the results of centralized two different manufacturer models are given below.

$$b^* = -\frac{(\Delta + 2c)\left[\alpha(2\beta + 2s + \varphi - 1) + r(-2\beta + 2s + \varphi - 1)\right] + c(\Delta + c)(3\alpha - r)}{\varphi(r + \alpha)(\Delta + 2c)} \tag{14}$$

$$\left(q_1^*, q_2^*\right) = \left(\frac{1}{2}\frac{(\Delta + c)(r + \alpha)}{(c + \beta)(\Delta + c) + c\beta}, \frac{1}{2}\frac{c(r + \alpha)}{(c + \beta)(\Delta + c) + c\beta}\right) \tag{15}$$

$$\Pi_{up}^* = \frac{1}{8}\frac{(\Delta + 2c)^2(\alpha + r)}{(c + \beta)(\Delta + c) + c\beta} \tag{16}$$

$$\left(\Pi_{d_1}^*, \Pi_{d_2}^*\right) = \left(\frac{1}{4}\frac{(\Delta + c)(r - \alpha)(r + \alpha)}{(c + \beta)(\Delta + c) + c\beta}, \frac{1}{4}\frac{c(r - \alpha)(r + \alpha)}{(c + \beta)(\Delta + c) + c\beta}\right) \tag{17}$$

Proposition 1. In the centralized manufacturer model, no matter under the uniform charging scheme or block charging scheme, as long as the potential market size of the upstream B2B market is large enough ($\alpha > \frac{r}{2}$), the contract variable of the scarce components will decrease with the increment of the production cost in the small-scale manufacturer.

3.2 Decentralized Manufacturers

We decentralize two manufacturers and each manufacturer pursues its own maximum profit, $\prod_{d_1}$ and $\prod_{d_2}$. The total ordering quantity is given by $Q = q_1 + q_2$.

3.2.1 Uniform Charging Scheme

The production cost function for the manufacturers here is quadratic $\frac{cq_i^2}{2}$. We let $\prod_{d_1}$ and $\prod_{d_2}$ be the profit of the large-scale and small-scale manufacturers as shown in (18) and (19) respectively, where $P_m = r - sQ$ and $Q = q_1 + q_2$.

$$\max_{q_1} \Pi_{d_1} = \left[P_m - \varphi(a_1 + bQ) - (1 - \varphi)(a_2 - Q)\right]q_1 - \frac{cq_1^2}{2} \tag{18}$$

$$\max_{q_2} \Pi_{d_2} = \left[P_m - \varphi(a_1 + bQ) - (1 - \varphi)(a_2 - Q)\right]q_2 - \frac{(c + \Delta)q_2^2}{2} \tag{19}$$

The summation of the profit functions in (19) and (20) are the expected revenue minus the expected cost of the component bought from the supplier and minus the production cost.

Lemma 3. Given the contract variable b of the price-quantity contract, if $c > -2\left[\varphi(b + 1) + s - 1\right]$ and $c + \Delta > -2\left[\varphi(b + 1) + s - 1\right]$ are satisfied, then the best responses of the large-scale and small-scale manufacturers under the uniform charging scheme are:

$$q_1^* = \frac{(\gamma-\alpha)(-1+s+\varphi+b\varphi+c+\Delta)}{-(-1+s+\varphi+b\varphi)^2+(-2+2s+2\varphi+2b\varphi+c)(-2+2s+2\varphi+2b\varphi+c \quad \Delta)} \tag{20}$$

$$q_2^* = \frac{(\gamma-\alpha)(-1+s+\varphi+b\varphi+ \quad)}{-(-1+s+\varphi+b\varphi)^2+(-2+2s+2\varphi+2b\varphi+ \quad)(-2+2s+2\varphi+2b\varphi+c \quad \Delta)} \tag{21}$$

where $\alpha = \varphi a_1 + (1-\varphi)a_2$.

Revisiting the first stage, the supplier proposes the contract with the different forms of contract variables of the price-quantity contract (b and -1) and anticipates that both of the manufacturers would respond accordingly to the decision from the supplier by maximizing the manufacturers' own profits (choosing their own best responses, q_1^* and q_2^*). The supplier's expected profit function is given by

$$\max_b \Pi_{up} = \varphi\left[(a_1 + bQ)Q - \frac{c_s Q^2}{2}\right] + (1-\varphi)\left[(a_2 - Q)Q - \frac{c_{ns} Q^2}{2}\right], \tag{20}$$

where $Q = q_1 + q_2$ is the total ordering quantity from two different manufacturers. The above objective function is the expected revenue minus the production cost with the probability of a supply chain disruption.

3.2.2 Block Charging Scheme

In this study, a backward induction is applied to solve the Stackelberg-type model. Similarly Π_{d_1} is the profit for the large-scale manufacturer with a small production cost c while Π_{d_2} is the profit for the small-scale manufacturer with a large production cost $c+\Delta$. The profits for the manufacturers are as follows, where $P_m = r - sQ$ and $Q = q_1 + q_2$.

$$\max_{q_1} \Pi_{d_1} = \left[P_m - \varphi\frac{(2a_1 + bQ)}{2} - (1-\varphi)\frac{(2a_2 - Q)}{2}\right]q_1 - \frac{cq_1^2}{2} \tag{21}$$

$$\max_{q_2} \Pi_{d_2} = \left[P_m - \varphi\frac{(2a_1 + bQ)}{2} - (1-\varphi)\frac{(2a_2 - Q)}{2}\right]q_2 - \frac{(c+\Delta)q_2^2}{2} \tag{22}$$

The summation of the profit functions in (23) and (24) are the expected revenue minus the expected cost of the component bought from the supplier and minus the production cost.

Lemma 4. Given the contract variable (b) of the price-quantity contract, if

$c > -\left[\varphi(b+1)+2s-1\right]$ and $c+\Delta > -\left[\varphi(b+1)+2s-1\right]$ are satisfied, then the block charging scheme are:

$$q_1^* = \frac{2(r-\alpha)(-1+2c+2s+2\Delta+\phi+b\phi)}{4c^2+4c(-2+4s+\Delta+2\phi+2b\phi)+(-1+2s+\phi+b\phi)(-3+6s+4\Delta+3\phi+3b\phi)} \tag{23}$$

$$q_2^* = \frac{2(r-\alpha)(-1+2c+2s+\phi+b\phi)}{4c^2+4c(-2+4s+\Delta+2\phi+2b\phi)+(-1+2s+\phi+b\phi)(-3+6s+4\Delta+3\phi+3b\phi)} \tag{24}$$

The supplier proposes the contract and anticipates that both of the manufacturers would respond accordingly to the decision from the supplier by maximizing the manufacturers' own profits (choosing their own best responses, q_1^* and q_2^*). The supplier's expected profit function is given by:

$$\max_b \Pi_{up} = \varphi\left[\frac{(2a_1+bQ)}{2}Q - \frac{c_s Q^2}{2}\right] + (1-\varphi)\left[\frac{(2a_2-Q)}{2}Q - \frac{c_{ns}Q^2}{2}\right] \tag{25}$$

where $Q = q_1 + q_2$ is the total ordering quantity from two different manufacturers.

4. Multiple Identical Manufacturers

4.1 Centralized Manufacturers

In the multiple identical manufacturers model, all multiple manufacturers are assumed to be symmetric. We focus on the centralized manufacturers, with a central planner responsible for optimizing the manufacturers' overall performances ($\sum_{i=1}^{n}\Pi_{d_i}$) as shown in Fig. 4. Considering the symmetric case, $q = q_i$ for all *i*, the total ordering quantity of the manufacturers is given by $Q = \sum_{i=1}^{n} q_i = nq$.

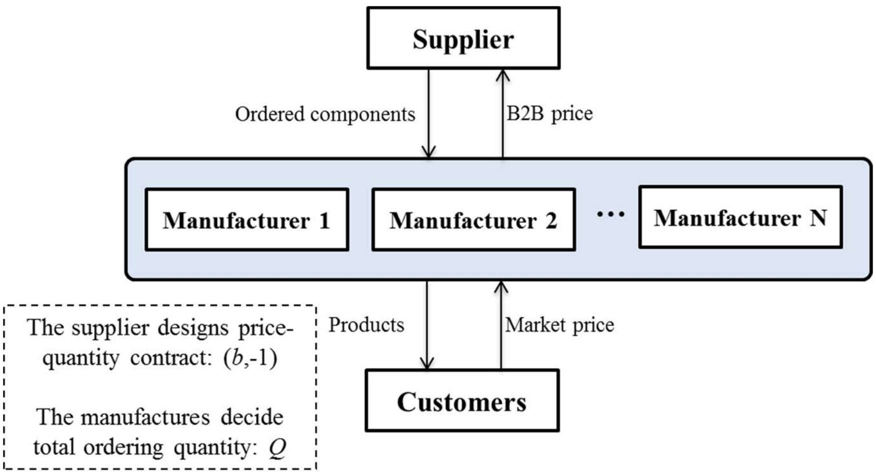

Fig. 4. The supply chain with the centralized manufacturers

4.1.1 Uniform Charging Scheme

As assumed in Section 2, the production cost function for the manufacturers used in this study is assumed to be $\frac{cq_i^2}{2}$ for simplicity. Applying the concepts of the expected value and uniform charging scheme, the expected total profit for the centralized manufacturers is given by:

$$\max_{q_i} \sum_{i=1}^{n} \Pi_{d_i} = \sum_{i=1}^{n} \left\{ \left[P_m - \varphi \left(a_1 + bQ \right) - (1-\varphi)\left(a_2 - Q \right) \right] q_i - \frac{cq_i^2}{2} \right\}, \tag{26}$$

where $P_m = r - sQ$. The above summation in the objective function is the expected revenue minus the expected cost of the components bought from the supplier and minus the production cost of the products sold to customers.

Lemma 5. Given the contract variable (b) of the price-quantity contract, if $c > -2\left[\varphi\left(b+1\right)+s-1\right]$, then the best response of the manufacturers under the uniform charging scheme is:

$$q_i^* = -\frac{\alpha - r}{2ns + 2\varphi bn + 2\varphi n + c - 2n}, \tag{27}$$

where $\alpha = \varphi a_1 + \left(1-\varphi\right) a_2$.

We have the relationship of the ordering quantity for each manufacturer (q_i^*) at the second stage and then moving back to the first stage is used to solve the optimal decision for the supplier. Similarly, the production cost for the supplier is assumed to be quadratic. The supplier's expected profit function is given by:

$$\max_{b} \Pi_{up} = \varphi \left[\left(a_1 + bQ \right) Q - \frac{c_s Q^2}{2} \right] + \left(1-\varphi\right) \left[\left(a_2 - Q \right) Q - \frac{c_{ns} Q^2}{2} \right], \tag{28}$$

where $Q = nq$ is the total ordering quantity from all manufacturers. The above objective function is the expected revenue minus the production cost with the probability of a supply chain disruption.

The supplier proposes the contract with different forms of contract variables of the price-quantity contract (*b* and -1) and anticipates that all manufacturers would respond accordingly to the decision from the supplier by maximizing the manufacturers' own profits (choose their own best responses, q_i^*).

Corollary 3. Under the uniform charging scheme, the results of the centralized multiple identical manufacturer model are given below:

$$b^* = -\frac{2n\left[(\varphi + s - 1)(\alpha + r) + \beta(\alpha - r)\right] - c(r - 3\alpha)}{2n\varphi\left(\alpha + r\right)} \tag{29}$$

$$q_i^* = \frac{1}{2}\frac{\alpha + r}{n\beta + c} \tag{30}$$

$$\Pi_{up}^* = \frac{1}{8}\frac{n\left(\alpha + r\right)^2}{n\beta + c} \tag{31}$$

$$\Pi_{d_i}^* = -\frac{1}{4}\frac{\left(\alpha + r\right)\left(\alpha - r\right)}{n\beta + c} \tag{32}$$

4.1.2 Block Charging Scheme

Here we discuss the block charging scheme, with a central planner responsible for optimizing the manufacturers' overall performances, $\sum_{i=1}^{n}\Pi_{d_i}$. We solve the two-stage sequential game by applying a backward induction.

Considering the symmetric case, we let $Q = \sum_{i=1}^{n} q_i = nq$ be the total ordering quantity of the manufacturers where $q = q_i$ and let $c = c_i$ be the production cost for each manufacturer *i* respectively. The expected total profit for the centralized manufacturers in the block charging scheme is given by:

$$\max_{q_i} \sum_{i=1}^{n} \Pi_{d_i} = \sum_{i=1}^{n} \left\{\left[P_m - \varphi\left(\frac{2a_1 + bQ}{2}\right) - \left(1 - \varphi\right)\left(\frac{2a_2 - Q}{2}\right)\right]q_i - \frac{cq_i^2}{2}\right\}), \tag{33}$$

where $P_m = r - sQ$.

Lemma 6. Given the contract variable b of the price-quantity contract, if $c > -\varphi(b+1) - 2s + 1$, then the best response of the manufacturers under the block charging scheme is:

$$q_i^* = -\frac{\alpha - r}{2ns + \varphi bn + \varphi n + c - n} \tag{34}$$

$$\text{where } \alpha = \varphi a_1 + (1-\varphi) a_2 .$$

Similarly, we move backward to the supplier's decision. The supplier's expected profit function at the first stage is

$$\max_b \Pi_{up} = \varphi\left[\left(\frac{2a_1 + bQ}{2}\right)Q - \frac{c_s Q^2}{2}\right] + (1-\varphi)\left[\left(\frac{2a_2 - Q}{2}\right)Q - \frac{c_{ns} Q^2}{2}\right], \quad (35)$$

where $Q = nq$ is the total ordering quantity from all manufacturers.

The supplier proposes the contract with different forms of contract variables of the price-quantity contract (b and -1) and anticipates that all manufacturers would respond accordingly to the decision from the supplier by maximizing the manufacturers' own profits (choose their own best responses, q_i^*).

Corollary 4. Under the block charging scheme, the results of the centralized multiple identical manufacturer model are given:

$$b^* = -\frac{n\left[(2s+\varphi-1)(\alpha+r)+2\beta(\alpha-r)\right]-c(r-3\alpha)}{n\varphi(\alpha+r)} \quad (36)$$

$$q_i^* = \frac{1}{2}\frac{\alpha+r}{n\beta+c} \quad (37)$$

$$\Pi_{up}^* = \frac{1}{8}\frac{n(\alpha+r)^2}{n\beta+c} \quad (38)$$

$$\Pi_{d_i}^* = -\frac{1}{4}\frac{(\alpha+r)(\alpha-r)}{n\beta+c} \quad (39)$$

Proposition 2. In the centralized manufacturer model, (i) the uniform charging scheme returns the identical ordering quantity and profits for both supplier and manufacturers to those under the block charging scheme, but (ii) the contract variables are different under the uniform and block charging schemes.

In the centralized multiple identical manufacturer model, we may have an intuition that the supplier earns more profit by adopting the uniform charging scheme because the manufacturers are required to pay a high price (a high b^*) even though they are acquiring a low volume (Q) of key components. Proposition 2 shows the counter intuition that the uniform and block charging schemes return the same ordering quantity for each manufacturer (q_i^*) and profits for both supplier and manufacturers (Π_{up}^* *and* Π_d^*). Proposition 2 implies that the supplier, acting as a leader, manipulates the contract variable (*b*) so that the contract variables are not the same, but the ordering quantity and profits for both supplier and manufacturers remain the same.

Proposition 3. In the centralized manufacturer model, the manufacturers would accept the contract only if the potential market size of the final market is larger than that of the upstream B2B market, i.e., $r > \alpha$.

The implication of $r > \alpha$ shows the potential market size of the final market is larger than that of the upstream B2B market, allowing manufacturers to make profits under the contract.

Proposition 4. In the centralized manufacturer model, as long as the potential market size of the upstream B2B market is large enough ($\alpha > \frac{r}{3}$), the contract variable of the scarce components will increase with the number of the manufacturers.

A large enough upstream B2B market offers opportunities for firms including supplier and manufacturers. Hence, the supplier's profit grows as the number of manufacturers increases, and the contract variable (*b*) also increases.

Proposition 5. In the centralized manufacturer model, (i) as the number of the manufacturers increases, the ordering quantity for each manufacturer falls. (ii) as the number for the manufacturers increases, the total ordering quantity rises.

It is intuitive that with the number of the manufacturers the total ordering quantity grows but the ordering quantity for each manufacturer decreases.

Proposition 6. In the centralized manufacturer model, we see that the increase in the number of the manufacturers leads to an increase in the supplier's profit under both uniform and block charging schemes and that an increase in the number of manufacturers results in a decrease in each manufacturer's profit.

As the total ordering quantity increases, the supplier sells more components and the average cost decreases. In addition, the supplier's profit increases. On the other hand, with more manufacturers, the total supply of final products will be larger and the market price will fall, thus leading to a drop in each manufacturer's profit.

4.2 Decentralized Manufacturers

Next to the centralized-manufacturer case, another widely discussed case involves the decentralized manufacturers as shown in Fig. 5. In the case of decentralized manufacturers, each manufacturer maximizes its own profit (Π_{d_i}), thereby allowing us to apply the concept of the Cournot competition among the manufacturers. All manufacturers, not cooperating with one another, sell homogeneous products to customers. Considering the symmetric case, the production cost $c = c_i$ for all manufacturers, but the total ordering quantity in the decentralized case is $Q = \sum_{i=1}^{n} q_i = q_i + \sum_{j \neq i} q_j$, where q_j is the other manufacturers except for manufacturer *i*.

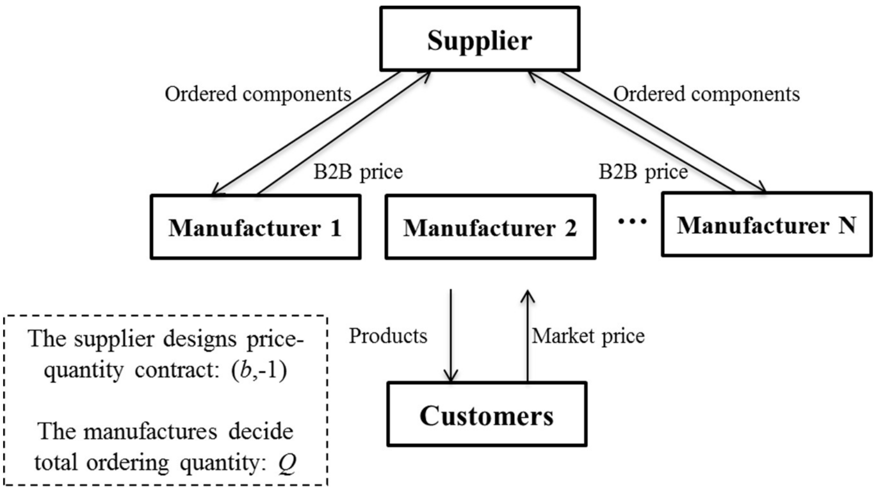

Fig. 5. The supply chain with the decentralized manufacturers

4.2.1 Uniform Charging Scheme

Now we examine the decentralized manufacturer case with the uniform charging scheme. The expected profit for each decentralized manufacturer *i* at the second stage is given by:

$$\max_{q_i} \Pi_{d_i} = \left(P_m - \varphi\left(a_1 + bQ\right) - \left(1-\varphi\right)\left(a_2 - Q\right)\right)q_i - \frac{cq_i^2}{2}, \tag{40}$$

where $P_m = r - sQ$.

Lemma 7. Given the contract variable *(b)* of the price-quantity contract, if $c > -2\left[\varphi\left(b+1\right) + s - 1\right]$, then the best response of the manufacturers under the uniform charging scheme is:

$$q_i^* = -\frac{\alpha - r}{\left(n+1\right)\left[\varphi\left(b+1\right) + s - 1\right] + c}, \tag{41}$$

where $\alpha = \varphi a_1 + \left(1-\varphi\right)a_2$.

We have the relationship of the ordering quantity for each manufacturer (q_i^*) at the second stage and then we return to the first stage to solve the optimal decision for the supplier. The expected profit for the supplier is

$$\max_{b} \Pi_{up} = \varphi\left[\left(a_1 + bQ\right)Q - \frac{c_s\left(nq\right)^2}{2}\right] + \left(1-\varphi\right)\left[\left(a_2 - Q\right)Q - \frac{c_{ns}\left(nq\right)^2}{2}\right], \tag{42}$$

where $Q^* = nq$ is the total ordering quantity from all manufacturers.

The supplier proposes the contract with different forms of contract variables of the price-quantity contract (b and -1) and anticipates that all manufacturers would respond accordingly to the decision from the supplier by maximizing the manufacturers' own profits (choosing their own best responses, q_i^*).

Corollary 5. Under the uniform charging scheme, the results of the multiple identical manufacturer model are given below.

$$b^* = -\frac{\left\{\begin{array}{l}\alpha(\beta+s)(n+1)^2 + n(\alpha - r)\left[(n+1)(\beta - s) + c\right] \\ +(n+1)\left[\alpha(c-1) - nr - n\varphi(\alpha - r)\right]\end{array}\right\}}{\varphi\left(n+1\right)\left(\alpha + nr\right)} \tag{43}$$

$$q_i^* = \frac{\alpha + nr}{n\left[\left(n+1\right)\beta + 2c\right]} \tag{44}$$

$$\Pi_{up}^* = \frac{1}{2}\frac{\left(\alpha + nr\right)^2}{\left(n+1\right)\left[\left(n+1\right)\beta + 2c\right]} \tag{45}$$

$$\Pi_{d_i}^* = \frac{1}{2}\frac{\left(\alpha + nr\right)\left[n(\alpha - r)(-2\beta(n+1) - 3c) - c(\alpha - n^2 r)\right]}{n^2\left(n+1\right)\left[\beta\left(n+1\right) + 2c\right]^2} \tag{46}$$

4.2.2 Block Charging Scheme

Similar to Section 3.2.2, we solve the two-stage sequential game by applying the backward induction. The block charging scheme's expected profit for each decentralized manufacturer i at the second stage is:

$$\max_{q_i} \Pi_{d_i} = \left[P_m - \varphi\left(\frac{2a_1 + bQ}{2}\right) - (1-\varphi)\left(\frac{2a_2 - Q}{2}\right)\right] q_i - \frac{cq_i^2}{2}, \tag{47}$$

where $P_m = r - sQ$.

Lemma 8. Given the contract variable *(b)* of the price-quantity contract, if $c > -\varphi(b+1) - 2s + 1$, then the best response of the manufacturers under the block charging scheme is:

$$q_i^* = -\frac{2(\alpha - r)}{(n+1)\left[\varphi(b+1) + 2s - 1\right] + 2c} \tag{48}$$

where $\alpha = \varphi a_1 + (1-\varphi)a_2$.

Moving back to the first stage, the expected profit for the supplier is

$$\max_{b} \Pi_{up} = \varphi\left[\left(\frac{2a_1 + bQ}{2}\right)Q - \frac{c_s(nq)^2}{2}\right] + (1-\varphi)\left[\left(\frac{2a_2 - Q}{2}\right)Q - \frac{c_{ns}(nq)^2}{2}\right], \tag{49}$$

where $Q^* = nq$ is the total ordering quantity from all manufacturers.

Corollary 6. Under the block charging scheme, the results of decentralized multiple identical manufacturer model are given:

$$b^* = -\frac{\left\{\begin{array}{l} 2\alpha s(n+1)^2 + 2n(\alpha - r)\left[(n+1)(\beta - s) + c\right] \\ +(n+1)\left[\alpha(2c-1) - nr + \varphi(\alpha + nr)\right] \end{array}\right\}}{\varphi(n+1)(\alpha + nr)} \tag{50}$$

$$q_i^* = \frac{\alpha + nr}{n\left[(n+1)\beta + 2c\right]} \tag{51}$$

$$\Pi_{up}^* = \frac{1}{2}\frac{(\alpha + nr)^2}{(n+1)\left[(n+1)\beta + 2c\right]} \tag{52}$$

$$\Pi_{d_i}^* = \frac{1}{2}\frac{(\alpha + nr)\left[n(\alpha - r)(-2\beta(n+1) - 3c) - c(\alpha - n^2 r)\right]}{n^2(n+1)\left[\beta(n+1) + 2c\right]^2} \tag{53}$$

Proposition 7. In the decentralized manufacturer model, (i) the uniform charging scheme returns the identical ordering quantity and profits for both supplier and manufacturers as those under the block charging scheme, but (ii) the contract variables are different under the uniform and block charging schemes.

The supplier, acting as a leader, manipulates the contract variable (b) so that the contract variables are not the same, but the ordering quantity and profits for both supplier and manufacturers remain the same.

Proposition 8. In the decentralized manufacturer model, (i) as the number of manufacturer increases, the ordering quantity for each manufacturer decreases (ii) and as the number of the manufacturers increases, the total ordering quantity also increases.

It becomes obvious that as the total ordering quantity by manufacturers grows, the ordering quantity for each manufacturer decreases.

5. Conclusions

Optimal supply chain performance has been extensively studied over the past years. In this thesis, we present the Stackelberg-type models with or without Cournot competition under the different charging schemes and centralized/decentralized mechanisms of the manufacturers. The objective of this thesis is to investigate the making of a contract related to dealing with potentially scarce components by discussing two types of models, namely the multiple identical manufacturers model and the two different manufacturers model

The analytical results show that in the multiple-identical-manufacturer models of the centralized and decentralized cases, the uniform and block charging schemes return the same profits for the supplier and manufacturers. The more manufacturers there are, the more likely are the profits for the suppliers; the more manufacturers, the less profits for each manufacturer. In the centralized-multiple-identical models as long as the potential market size of the B2B market is large enough, the contract variable of the scarce components increases with the number of the manufacturers. In the centralized-two-different-manufacturers models as long as the potential market size of the B2B market is large enough, the contract variable of the scarce components decreases with the incremental production cost of the small-scale manufacturer. We also conclude from the numerical studies that in the multiple identical manufacturer model the supplier gains more profits than in the centralized manufacturer case.

In the two different manufacturers model, the scenario of the centralized manufacturer case under the block charging scheme outperforms all the other scenarios.

In summary, the results presented in this study provide a supplier with valuable insights into designing a contract applicable to the potentially scarce component situation and to the probability of a supply chain disruption. There are several potential future research opportunities. One future research direction is to investigate the contract design in a broader supply chain context. It will be interesting to extend the supply chain from a two-tier to a multi-tier supply chain problem, with demand uncertainty in the final market. Another possible extension is to consider shortage costs, opportunities lost in production and business activities caused by failure to meet market demand.

References

Alonso, E., Gregory, J., Field, F., & Kirchain, R. (2007). Material availability and the supply chain: risks, effects, and responses. *Environmental science &* technology, *41*(19), 6649-6656.

Ang, E., Iancu, D. A., & Swinney, R. (2016). Disruption risk and optimal sourcing in multitier supply networks. *Management Science*, 63(8), 2397-2419.

Banal-Estañol, A., & Rupérez Micola, A. (2011). Behavioural simulations in spot electricity markets. *European Journal of Operational Research*, *214*(1), 147-159.

Boland, J., & Whittington, D. (2000). Water tariff design in developing countries: Disadvantages of increasing block tariffs and advantages of uniform price with rebate designs. *IDRC Research Paper*.

Cachon, G. P., & Lariviere, M. A. (2005). Supply chain coordination with Revenue-sharing contracts: Strengths and limitations. *Management Science*, *51*(1), 30-44.

Chen, X., (2015). A model of trade credit in a capital-constrained distribution channel. *International Journal of Production Economics*, *159*, 347-357.

Chen, K., & Xiao, T. (2009). Demand disruption and coordination of the supply chain with a dominant retailer. *European Journal of Operational Research*, *197*(1), 225-234.

Chopra, S., & Sodhi, M. S. (2012). Managing risk to avoid supply-chain breakdown. *MIT Sloan Management Review*, *46*(1), 53-61.

De Giovanni, P., (2018). A joint maximization incentive in closed-loop supply chains with competing retailers: The case of spent-battery recycling. *European Journal of Operational Research*, *268*(1), 128-147.

El Ouardighi, F., & Erickson, G., (2015). Production capacity buildup and double marginalization mitigation in a dynamic supply chain. *Journal of the Operational Research Society*, *66*(8), 1281-1296.

Ghadge, A., Dani, S., Ojha, R., & Caldwell, N., (2017). Using risk sharing contracts for supply chain risk mitigation: A buyer-supplier power and dependence perspective. *Computers & Industrial Engineering*, *103*, 262-270.

Hendricks, K. B., & Singhal, V. R. (2005). An empirical analysis of the effect of supply chain disruptions on long-run stock price performance and equity risk of the firm. *Production and Operations management*, *14*(1), 35-52.

Hong, I-H., Chang, P.-Y. (2018). The transaction contract of scarce materials in a supply chain: Paying a higher price to acquire a higher quantity. submitted to *Journal of the Operational Research Society.*

Hou, J., Zeng, A. Z., & Zhao, L. (2010). Coordination with a backup supplier through buy-back contract under supply disruption. *Transportation Research Part E: Logistics and Transportation Review*, *46*(6), 881-895.

Lau, A. H., Lau, H-S., & Wang, J-C., (2007). Some properties of buyback and other related schemes in a newsvendor-product supply chain with price-sensitive demand. *Journal of the Operational Research Society*, *58*(4), 491-504.

Lian, Z., Deshmukh, A. (2009). Analysis of supply contracts with quantity flexibility. *European Journal of Operational Research*, *196*(2), 526-533.

Li, X., & Wang, Q. (2007). Coordination mechanisms of supply chain systems. *European journal of operational research*, *179*(1), 1-16.

McClure, J., & Kumcu, E. (2008). Promotions and product pricing: Parsimony versus Veblenesque demand. *Journal of Economic Behavior & Organization*, *65*(1), 105 -117.

Olson, M., Rassenti, S., Rigdon, M., & Smith, V. (2003). Market design and human trading behavior in electricity markets. *IIE Transactions*, *35*(9), 833-849.

Phillips, D. C. (Ed.). (2014). *Encyclopedia of Educational Theory and Philosophy*. SAGE Publications.

Poppo, L., Zhou, K. Z., & Li, J. J., (2016). When can you trust "trust"? Calculative trust, relational trust, and supplier performance. *Strategic Management Journal*, *37*(4), 724-741.

Salema, M. I. G., Barbosa-Povoa, A. P., & Novais, A. Q. (2007). An optimization model for the design of a capacitated multi-product reverse logistics network with uncertainty. *European Journal of Operational Research*, *179*(3), 1063-1077.

Srinivasan, M., Mukherjee, D., & Gaur, A. S. (2011). Buyer–supplier partnership quality and supply chain performance: Moderating role of risks, and environmental uncertainty. *European Management Journal*, *29*(4), 260-271.

Taylor, T. A. (2002). Supply chain coordination under channel rebates with sales effort effects. *Management Science*, *48*(8), 992-1007.

Taylor, T. A., & Xiao, W. (2009). Incentives for retailer forecasting: Rebates vs. returns. *Management Science*, *55*(10), 1654-1669.

Tomlin, B. (2006). On the value of mitigation and contingency strategies for managing supply chain disruption risks. *Management Science*, *52*(5), 639-657.

Tomlin, B. (2009). Disruption-management strategies for short life-cycle products. *Naval Research Logistics (NRL)*, *56*(4), 318-347.

Wang, Y., Jiang, L., & Shen, Z. J. (2004). Channel performance under consignment contract with revenue sharing. *Management Science*, *50*(1), 34-47.

Won, Y., (2017). How can a retailer identify the dominant strategy in a newsvendor situation with a supplier take-it-or-leave-it offer?. *International Transactions in Operational Research*, *24*(6), 1435-1462.

Wong, W. K., Qi, J., & Leung, S. Y. S. (2009). Coordinating supply chains with sales rebate contracts and vendor-managed inventory. *International Journal of Production Economics*, *120*(1), 151-161.

Appendix

Proof of Lemma 1.

To determine that expected profit function (2) is concave requires showing that its Hessian is negative semi-definite (concavity condition of $\prod d_1 + \prod d_2$ in q_1 and q_2). The Hessian matrix is as follows.

$$H(q_1, q_2) = \begin{pmatrix} -2b\varphi - 2\varphi - 2s + 2 - c, -2b\varphi - 2\varphi - 2s + 2 \\ -2b\varphi - 2\varphi - 2s + 2, -2b\varphi - 2\varphi - 2s + 2 - c - \Delta \end{pmatrix}$$

The expected profit function (2) is maximized as the optimal ordering quantities for both manufacturer (q_1^* and q_2^*) from the first-order condition are satisfied under the concavity condition of $\prod d_1 + \prod d_2$ in q_1 and q_2.

$$q_1^* = -\frac{\Delta a_1\varphi - \Delta a_2\varphi + a_1 c\varphi - a_2 c\varphi \quad \Delta a_2 - \Delta r + a_2 c - cr}{2\Delta b\varphi + 4bc\varphi + \Delta c + 2\Delta\varphi + 2\Delta s + c^2 + 4c\varphi + 4cs - 2\Delta - 4c}$$

$$q_2^* = -\frac{(a_1\varphi - a_2\varphi + a_2 - r)c}{2\Delta b\varphi + 4bc\varphi + \Delta c + 2\Delta\varphi + 2\Delta s + c^2 + 4c\varphi + 4cs - 2\Delta - 4c}$$

■

Proof of Lemma 2.

To determine that expected profit function (10) is concave requires showing that its Hessian is negative semi-definite (concavity condition of $\prod d_1 + \prod d_2$ in q_1 and q_2). The Hessian matrix is as follows.

$$H(q_1, q_2) = \begin{pmatrix} -b\varphi - \varphi - 2s + 1 - c, -b\varphi - \varphi - 2s + 1 \\ -b\varphi - \varphi - 2s + 1, -b\varphi - \varphi - 2s + 1 - c - \Delta \end{pmatrix}$$

The expected profit function (10) is maximized as the optimal ordering quantities for the both manufacturers (q_1^* and q_2^*) from the first-order conditions are satisfied under the concavity condition of $\prod d_1 + \prod d_2$ in q_1 and q_2.

$$q_1^* = -\frac{\Delta a_1\varphi - \Delta a_2\varphi + a_1 c\varphi - a_2 c\varphi \quad \Delta a_2 - \Delta r + a_2 c - cr}{\Delta b\varphi + 2bc\varphi + \Delta c + \Delta\varphi + 2\Delta s + c^2 + 2c\varphi + 4cs - \Delta - 2c}$$

$$q_2^* = -\frac{(a_1\varphi - a_2\varphi + a_2 - r)c}{\Delta b\varphi + 2bc\varphi + \Delta c + \Delta\varphi + 2\Delta s + c^2 + 2c\varphi + 4cs - \Delta - 2c}$$

■

Proof of Lemma 3.

Take the second partial derivative of both expected profit function (18) and (19) with respect to q_1 and q_2 respectively.

$$\frac{\partial^2 \prod_{d_1}}{\partial q_1^2} = -2b\varphi - 2\varphi - 2s - c + 2$$

$$\frac{\partial^2 \prod_{d_2}}{\partial q_2^2} = -2b\varphi - 2\varphi - 2s - c - \Delta + 2$$

We then have the concavity condition from the second order condition as follows.

$$\mathrm{c} > -2[\varphi(b+1) + s - 1]$$

$$\mathrm{c} + \Delta > -2[\varphi(b+1) + s - 1]$$

The expected profit function (18) is maximized as the optimal ordering quantity for the large-scale manufacturer (q_1^*) from the first-order condition is satisfied under the concavity condition of $\prod d_1$ in q_1.

$$q_1^* = -\frac{[\varphi a_1 + (1-\varphi)a_2 - \gamma](\varphi b + \varphi + s + c + \Delta - 1)}{2(\varphi b + \varphi + s)(2c + \Delta - 3) + 3\varphi^2 b(b+2) + 6\varphi s(b+1) + c(\Delta - 4) + 3(\varphi^2 + s^2 + 1) + c^2 - 2\Delta}$$

Similarly, the expected profit function (19) is maximized as the optimal ordering quantity for the small-scale manufacturer (q_2^*) from the first-order condition is satisfied under the concavity condition of $\prod d_2$ in q_2.

$$q_2^* = -\frac{\varphi[(a_1 - a_2)(\varphi b + \varphi + s + c) - (a_1 - 2a_2) + b(a_2 - r)] - r(\varphi + s + c - 1)}{2(\varphi b + \varphi + s)(2c + \Delta - 3) + 3\varphi^2 b(b+2) + 6\varphi s(b+1) + c(\Delta - 4) + 3(\varphi^2 + s^2 + 1) + c^2 - 2\Delta}$$

■

Proof of Lemma 4.

Take the second partial derivative of the expected profit function (23) and (24) with respect to q_1 and q_2 respectively.

$$\frac{\partial^2 \sum_{i=1}^{2} \prod_{d_i}}{\partial q_1^2} = -b\varphi - c - \varphi - 2s + 1$$

$$\frac{\partial^2 \sum_{i=1}^{2} \prod_{d_i}}{\partial q_2^2} = -b\varphi - \Delta - c - \varphi - 2s + 1$$

We then have the following concavity condition from the second order condition.

$$\mathrm{c} > -[\varphi(b+1) + 2s - 1]$$

$$c + \Delta > -[\varphi(b+1) + 2s - 1]$$

The expected profit function (23) is maximized as the optimal ordering quantity for the large-scale manufacturer (q_1^*) from the first-order condition is satisfied under the concavity condition of $\prod d_1$ in q_1. $q_2^* =$

$$-\frac{2[\varphi a_1+(1-\varphi)a_2-r](\varphi b+\varphi+2s+2c+2\Delta-1)}{2(\varphi b+\varphi+2\ )(4c+2\Delta-3)+3\varphi^2 b(b+2)+12\varphi\ (b+1)+c4(\Delta-2)+3(\varphi^2+4s^2+1)4(c^2-\Delta)}$$

Similarly, the expected profit function (24) is maximized as the optimal ordering quantity for the small-scale manufacturer (q_2^*) from the first-order condition is satisfied under the concavity condition of $\prod d_2$ in q_2.

$$q_2^* = -\frac{\varphi[(a_1-a_2)(\varphi b+\varphi+2s+2c)-(a_1-2a_2)+b(a_2-r)]-r(\varphi+2s+2c-1)}{2(\varphi b+\varphi+2\ )(4c+2\Delta-3)+3\varphi^2 b(b+2)+12\varphi s(b+1)+c4(\Delta-2)+3(\varphi^2+4s^2+1)4(c^2-\Delta)}$$

■

Proof of Lemma 5.

At the second stage of the Stackelberg game, take the second partial derivative of the expected profit function:

$$\max_{q_i} \sum_{i=1}^{n} \prod_{d_i} = \sum_{i=1}^{n} \{[P_m - \varphi(a_1 + bQ) - (1-\varphi)(a_2 - Q)]q_i - \frac{cq_i^2}{2}\}$$

with respect to q_1, q_2… and q_n and solve a set of simultaneous equations. From the second order condition, we have the optimal ordering quantity for each manufacturer *k*.

$$\frac{\partial^2 \sum_{i=1}^{n} \prod_{d_i}}{\partial q_k^2} = -2n(s + \varphi b + \varphi + \frac{c}{2} - 1)$$

Because of the symmetric case, we take the second partial derivative of the profit function with respect to q_i, allowing us to simply solve the simultaneous equations. Hence, the second-order condition is

$$\frac{\partial^2 \sum_{i=1}^{n} \prod_{d_i}}{\partial q_i^2} = -2n(s + \varphi b + \varphi + \frac{c}{2} - 1)$$

We have the concavity condition of $\sum_{i=1}^{n} \prod_{d_i}$ in q_i from the second order condition.

$$c > -2[\varphi(b+1) + s - 1]$$

Under the concavity condition of $\sum_{i=1}^{n} \prod_{d_i}$ in q_i, the expected profit function is maximized as the optimal ordering quantity for each manufacturer q_i^* from the first-order condition is satisfied.

$$q_i^* = -\frac{\alpha - r}{2ns + 2\varphi bn + 2\varphi n + c - 2n} \text{ ,}$$

where α he$a_1 + (1-\varphi)a_2$.

▓

Proof of Lemma 6.

We could simply take the partial derivative of the expected total profit for the centralized manufacturers in the block charging scheme:

$$\max_{q_i} \sum_{i=1}^{n} \prod_{d_i} = \sum_{i=1}^{n} \{\left[P_m - \varphi\left(\frac{2a_1 + bQ}{2}\right) - (1-\varphi)\left(\frac{2a_2 - Q}{2}\right)\right] q_i - \frac{cq_i^2}{2}\},$$

where $P_m = r - sQ$, with respect to q_i under the symmetric case. Therefore, the second-order condition is

$$\frac{\partial^2 \prod_d}{\partial q_i^2} = -2ns - \varphi bn - \varphi n - c + n$$

Then we have the following concavity condition of $\sum_{i=1}^{n} \prod_{d_i}$ in q_i from the second order condition.

$$c > -n[\varphi(b+1) + 2s - 1]$$

Under the concavity condition of $\sum_{i=1}^{n} \prod_{d_i}$ in q_i, the expected profit function is maximized as the optimal ordering quantity for each manufacturer (q_i^*) from the first-order condition is satisfied:

$$q_i^* = -\frac{\alpha - \gamma}{2ns + \varphi bn + \varphi n + c - n}, \text{ where } \alpha \;\; \varphi a_1 + (1-\varphi)a_2.$$

▓

Proof of Lemma 7.

Considering the symmetric case, we take the second order condition of the profit function $\prod_{d_i}$ with respect to q_i.

$$\frac{\partial^2 \prod_{d_i}}{\partial q_i^2} = -2(s + \varphi b + \varphi + \frac{c}{2} - 1)$$

Then the concavity condition of $\prod_{d_i}$ in q_i is

$$c > -2[\varphi(b+1) + s - 1]$$

Under the concavity condition of $\prod_{d_i}$ in q_i, the expected profit function (42) is maximized as the optimal ordering quantity for each manufacturer (q_i^*) from the first-order condition holds:

$$q_i^* = -\frac{(b\varphi+\varphi+s-1)\sum_{j\neq i} q_j + \alpha - \gamma}{2(b\varphi+\varphi+s-1)+c}\text{, where } \alpha = \varphi a_1 + (1-\varphi)a_2.$$

Considering the symmetric case, we let $q_j = q_i^*$ and $\sum_{j\neq i} q_j = (n-1)q_i^*$ for all the other manufacturer *j*. Then the ordering quantity for each manufacturer *i* is

$$q_i^* = -\frac{\alpha - \gamma}{(n+1)[\varphi(b+1)+s-1]+c}.$$

Proof of Lemma 8.

Considering the symmetric case, we take the second derivative of $\prod_{d_i}$ with respect to q_i as follows.

$$\frac{\partial^2 \prod_d}{\partial q_i^2} = -2s - \varphi b - \varphi - c + 1$$

Then we derive the concavity condition from the above second-order condition.

$$c > -\varphi(b+1) - s + 1$$

Under the concavity condition of $\prod_{d_i}$ in q_i, the expected profit for each of the decentralized manufacturer (49) is maximized as the optimal ordering quantity for each manufacturer (q_i^*) from the first-order condition is satisfied.

$$q_i^* = -\frac{q_j[(n-1)(b\varphi+\varphi+2s-1)] - 2(\alpha - r)}{2(b\varphi+\varphi+2s-1)+2c}$$

Considering the symmetric case, we let $q_j = q_i^*$ and $\sum_{j\neq i} q_j = (n-1)q_i^*$ for all the other manufacturer *j*. Consequently, the ordering quantity for each manufacturer is given by

$$q_i^* = -\frac{2(\alpha - r)}{(n+1)[\varphi(b+1)+2s-1]+2c}$$

■

Proof of Corollary 1.

Take the second-order condition of $\prod_{up}$ with respect to b and the concavity condition $\prod_{up}$ is as below.

$$\frac{\partial^2 \prod_{up}}{\partial b^2} = -\frac{\{(\Delta+2c)[\alpha(-2+3\beta+2\varphi+2s+2c)-r(2+3\beta-2\varphi-2s+c)]+c\Delta(2\alpha-r)\}}{\varphi(\alpha+r)(\Delta+2c)} < 0$$

From the first-order condition, the supplier's optimal contract variable under a supply chain disruption is:

$$b^* = -\frac{1}{2}\frac{\{2(\Delta+2c)[\alpha(-1+\beta+\varphi+s)-r(1+\beta-\varphi-s)]+c(\Delta+c)(3\alpha-r)\}}{\varphi(\alpha+r)(\Delta+2c)}.$$

Where $\alpha = \varphi a_1 + (1-\varphi)a_2$ and $\beta = \varphi c_s + (1-\varphi)c_{ns} + 2s$.

The concavity condition holds. Each of the manufacturer's optimal decision (q_1^* and q_2^*) under the concavity condition of $\prod_{up}$ in *b* is:

$$(q_1^*, q_2^*) = (\frac{1}{2}\frac{(\Delta+c)(\alpha+\gamma)}{(c+\beta)(\Delta+c)+c\beta}, \frac{1}{2}\frac{c(\alpha+r)}{(c+\beta)(\Delta+c)+c\beta}).$$

The optimal profit for the supplier is

$$\prod_{up}^* = \frac{1}{8}\frac{c(\alpha+r)(\alpha-r)}{(c+\beta)(\Delta+c)+c}$$

The optimal profit for each manufacturer is as follows:

$$(\prod_{d_1}^*, \prod_{d_2}^*) = (-\frac{1}{4}\frac{(\Delta+c)(\alpha+r)(\alpha-r)}{(c+\beta)(\Delta+c)+c}, -\frac{1}{4}\frac{c(\alpha+r)(\alpha-r)}{(c+\beta)(\Delta+c)+c})$$

▓

Proof of Corollary 2.

Take the second-order condition of $\prod_{up}$ with respect to b and the concavity condition $\prod_{up}$ is given below.

$$\frac{\partial^2 \prod_{up}}{\partial b^2} = -\frac{\{(\Delta+2c)[3\alpha(\beta+2s+c)-r(2+3\beta+2s-2\varphi)]\}+c\Delta(3\alpha-2r)-2cr(\Delta+2c)}{2\varphi r(\Delta+2c)} < 0$$

From the first-order condition, the supplier's optimal contract variable under a supply chain disruption is:

$$b^* = -\frac{\{(\Delta+2c)[\alpha(\beta+2s+c)-r(1+\beta-\varphi)]\}+c\Delta(\alpha-r)-c^2 r}{\varphi r(\Delta+2c)}$$

Where $\alpha = \varphi a_1 + (1-\varphi)a_2$ and $\beta = \varphi c_s + (1-\varphi)c_{ns} + 2s$.

The concavity condition holds. Each of the manufacturer's optimal decision (q_1^* and q_2^*) under the concavity condition of $\prod_{up}$ in b is:

$$(q_1^*, q_2^*) = (\frac{(\Delta+c)r}{(2s+\beta)(\Delta+2c)+2c(\Delta+c)}, \frac{cr}{(2s+\beta)(\Delta+2c)+2c(\Delta+c)}).$$

The optimal profit for the supplier is

$$\prod_{up}^* = \frac{1}{2}\frac{r^2(\Delta+2c)}{\{(2s+\beta)(\Delta+2c)+2c(\Delta+c)\}}$$

The optimal profit for each manufacturer is as follows:

$$(\prod_{d_1}^*, \prod_{d_2}^*) = (-\frac{1}{2}\frac{r(\Delta+c)(\alpha-r)}{\{(2s+\beta)(\Delta+2c)+2c(\Delta+c)\}}, -\frac{1}{2}\frac{r(\Delta+2c)(\alpha-r)}{\{(2s+\beta)(\Delta+2c)+2c(\Delta+c)\}})$$

▓

Proof of Corollary 3.

Move backward to the first stage of the game. Taking the second partial derivative of $\prod_{up}$ with respect to b yields the concave threshold for b as below; that is, if the following inequality holds, $\prod_{up}$ is concave in b.

$$\frac{\partial^2 \prod_{up}}{\partial b^2} = -\frac{2n\alpha(\varphi+s)+\alpha(3n\beta+4c-2n)-r(3n\beta+2c)-2nr(1-\varphi-s)}{2n\varphi(\alpha+r)} < 0$$

From the first-order condition, the supplier's optimal contract variable under a supply chain disruption is given below.

$$b^* = -\frac{2n[(s+\varphi-1)(\alpha+r)+\beta(\alpha-r)]-c(r-3\alpha)}{2n\varphi(\alpha+r)}.$$

where α h$\varphi a_1 + (1-\varphi)a_2$ and $\beta = \varphi c_s + (1-\varphi)c_{ns} + 2s$.

Satisfying the above concave threshold, the concavity verification is valid, indicating the supplier's decision in equilibrium is exactly the optimal solution. Each of the manufacturer's optimal decision is

$$q_i^* = \frac{1}{2}\frac{\alpha+\gamma}{n\beta+c}.$$

The optimal profit for the supplier is

$$\prod_{up}^{*} = \frac{1}{8}\frac{n(\alpha + r)^2}{n\beta + c}$$

Finally, we obtain the following optimal profit for each manufacturer.

$$\prod_{d_i}^{*} = -\frac{1}{4}\frac{(\alpha + r)(\alpha - r)}{n\beta + c}$$

▓

Proof of Corollary 4.

Move backward to the first stage and take the second derivative with respect to b, we have the concave threshold for b as follows.

$$\frac{\partial^2 \prod_{up}}{\partial b^2} = -\frac{n\varphi(\alpha + r) + \alpha(2ns + 4c - n) + 3n\beta(\alpha - r) - r(2c + n - 2ns)}{n\varphi(\alpha + r)} < 0$$

From the first-order condition, the supplier's optimal contract variable under a supply chain disruption is:

$$b^* = -\frac{n[(2s+\varphi-1)(\alpha+r)+\beta(\alpha-r)]-c(r-3\alpha)}{n\varphi(\alpha+r)}.$$

where α h$\varphi a_1 + (1-\varphi)a_2$ and $\beta = \varphi c_s + (1-\varphi)c_{ns} + 2s$.

The concavity verification is valid, indicating the supplier's decision in equilibrium is exactly the optimal solution. Each of the manufacturer's optimal decision is given by

$$q_i^* = \frac{1}{2}\frac{\alpha+\gamma}{n\beta+c}.$$

The optimal profit for the supplier is

$$\prod_{up}^{*} = \frac{1}{8}\frac{n(\alpha + r)^2}{n\beta + c}$$

We obtain the optimal profit for each manufacturer:

$$\prod_{d_i}^{*} = -\frac{1}{4}\frac{(\alpha + r)(\alpha - r)}{n\beta + c}$$

▓

Proof of Corollary 5.

Take the second partial derivative of $\prod_{up}$ with respect to b yields the concave threshold for b.

$$
b < \frac{\left\{\begin{matrix} \varphi(a_1-a_2)\{(n+1)[2(3s+1)(-s-c)-\varphi(3n+2)]\}-\varphi(n+1)(a_1-2a_2)(3c_{ns}n-1) \\ +3\varphi nr(n+1)(c_s-c_{ns})-(n+1)[3a_2n(c_s\varphi+c_{ns})-2n\ \ (1-\varphi)-3c_{ns}nr-2a_2]- \\ 2a_2s(n+1)(3n+1)-2c(3a_2n-2nr+a_2) \end{matrix}\right\}}{2\varphi(n+1)(\alpha+nr)}
$$

From the first-order condition, the supplier's optimal contract variable under a supply chain disruption is as follows.

$$
b^* = -\frac{\alpha(\beta+s)(n+1)^2+n(\alpha-r)[(n+1)(\beta-s)+c]+(n+1)[\alpha(c-1)-nr-n\ \ (\alpha-r)]}{\varphi(n+1)(\alpha+nr)}.
$$

where $\alpha = \varphi a_1 + (1-\varphi)a_2$ and $\beta = \varphi c_s + (1-\varphi)c_{ns} + 2s$.

Similarly, under the concavity condition of $\prod_{up}$ in b, each of the manufacturers' optimal decision is given by

$$
q_i^* = \frac{\alpha+nr}{n[(n+1)\beta+2c]}
$$

The optimal profit for the supplier is

$$
\prod_{up}^* = \frac{1}{2}\frac{(\alpha+nr)^2}{(n+1)[(n+1)\beta+2c]}
$$

We have the optimal profit for each manufacturer.

$$
\prod_{d_i}^* = \frac{1}{2}\frac{(\alpha+nr)[n(\alpha-r)(-2\beta(n+1)-3c)-c(\alpha-n^2r)]}{n^2(n+1)[\beta(n+1)+2c]^2}
$$

▓

Proof of Corollary 6.

When the concavity condition holds, the supplier's optimal contract variable under a supply chain disruption from the first-order condition is:

$$
b^* = -\frac{2\alpha s(n+1)^2+2n(\alpha-r)[(n+1)(\beta-s)+c]+(n+1)[\alpha(2c-1)-nr+\varphi(\alpha+nr)]}{\varphi(n+1)(\alpha+nr)}.
$$

where $\alpha = \varphi a_1 + (1-\varphi)a_2$ and $\beta = \varphi c_s + (1-\varphi)c_{ns} + 2s$

Each of the manufacturer's optimal decision (q_i^*) under the concavity condition of $\prod_{up}$ in b.

$$
q_i^* = \frac{\alpha+nr}{n[(n+1)\beta+2c]}
$$

The optimal profit for the supplier is

$$
\prod_{up}^* = \frac{1}{2}\frac{(\alpha+nr)^2}{(n+1)[(n+1)\beta+2c]}.
$$

The optimal profit for each manufacturer is as follows:

$$\prod_{d_i}^{*} = \frac{1}{2}\frac{(\alpha+nr)[n(\alpha-r)(-2\beta(n+1)-3c)-c(\alpha-n^2r)]}{n^2(n+1)[\beta(n+1)+2c]^2}$$

■

Proof of Proposition 1.

We take the first derivative of b^* under the uniform charging schemes as shown in (6) with respect to Δ, $\frac{\partial b^*}{\partial \Delta} = -\frac{1}{2}\frac{c^2(3\alpha-r)}{\varphi(\Delta+2c)^2(\alpha+r)}$. As long as $(3\alpha - r)$ is positive, $\frac{\partial b^*}{\partial \Delta}$ is negative. Hence b^* decreases as Δ increases. For convenience, the condition can be rewritten as:$\alpha > \frac{r}{3}$.

Similarly, we take the first derivative of b^* under the block charging schemes as shown in (14), $\frac{\partial b^*}{\partial \Delta} = -\frac{c^2(2\alpha-r)}{\varphi r(\Delta+2c)^2}$. As long as $(2\alpha - r)$ is positive, $\frac{\partial b^*}{\partial \Delta}$ is negative so that b^* decreases as Δ increases. For convenience, the condition can be rewritten as: $\alpha > \frac{r}{2}$.

■

Proof of Proposition 2.

As shown in (31)-(34) and (38)-(41), the proposition 1 follows.

■

Proof of Proposition 3.

Undoubtedly, only when the market is profitable for the manufacturers do the manufacturers accept the contract. Both of the profit for each manufacturer from (34) and (41) are $\prod_{d_i}^{*} = -\frac{1}{4}\frac{(\alpha+r)(\alpha-r)}{n\beta+c_i}$ where α he$a_1 + (1-\varphi)a_2$ and $\beta = \varphi c_s + (1-\varphi)c_{ns} + 2s$ are non-negative. As assumed, all parameters are positive; hence, $(\alpha - r)$ must be negative so that the manufacturers could gain profits.

■

Proof of Proposition 4.

We take the first derivative of b^* under both charging schemes (see (31) and (38)) with respect to *n* respectively. In terms of the uniform charging scheme, $\frac{\partial b^*}{\partial n} = \frac{1}{2}\frac{c_i(3\alpha-r)}{n^2\varphi(\alpha+r)}$. In terms of the block charging scheme, $\frac{\partial b^*}{\partial n} = \frac{c_i(3\alpha-r)}{n^2\varphi(\alpha+r)}$, we can see from the above-mentioned derivatives with respect to *n* that as long as $(3\alpha - r)$ is positive, $\frac{\partial b^*}{\partial n}$ is positive, showing that b^* increases as n increases. For convenience, the condition can be rewritten as: $\alpha > \frac{r}{3}$.

■

Proof of Proposition 5.

(i): From (32) and (39), the optimal ordering quantity for each manufacturer under two charging schemes is $q_i^* = \frac{1}{2}\frac{\alpha+r}{n\beta+c_i}$, where α $wa_1 + (1-\varphi)a_2$ and $\beta = \varphi c_s + (1-\varphi)c_{ns} + 2s$ are not related to n. As the number of the manufacturers increases, q_i^* decreases. (ii): In terms of the total ordering quantity $Q = \sum_{i=1}^{n} q_i = nq_i$ from (32) and (39), we take the first derivative with respect to n and then we have $\frac{\partial Q}{\partial n} = \frac{1}{2}\frac{(\alpha+\gamma)c}{(n\beta+c)^2}$. The first order condition is non-negative, showing an increase in n results in the increase in Q.

■

Proof of Proposition 6.

(i): Take the first derivative of $\prod_{up}^*$ from (32) and (40) with respect to n, $\frac{\partial \prod_{up}^*}{\partial n} = \frac{1}{8}\frac{(\alpha+\gamma)^2 c_i}{(n\beta+c_i)^2}$. Because of the quadratic terms in both denominator and numerator and positive production cost c_i, $\frac{\partial \prod_{up}^*}{\partial n}$ is always positive. The supplier's profit increases as n increases. (ii): Take the first derivative of $\prod_{d_i}^*$ from (32) and (40) with respect to n: $\frac{\partial \prod_{d_i}^*}{\partial n} = \frac{1}{4}\frac{(\alpha-\gamma)(\alpha+\gamma)\beta}{(n\beta+c_i)^2}$. As proved in Proposition 2, $\alpha - r < 0$ must be satisfied so that the manufacturers accept the contract with the supplier. Thus, $\frac{\partial \prod_{d_i}^*}{\partial n}$ is always negative and the profit for each manufacturer ($\prod_{d_i}^*$) decreases as n increases.

■

Proof of proposition 7.

As shown in (45)-(48) and (52)-(55), the proposition 7 follows.

■

Proof of Proposition 8.

(i): From (46) and (53), the optimal ordering quantity for each manufacturer under two charging schemes is $q_i^* = \frac{\alpha+nr}{n[(n+1)\beta+2c]}$ where α he$a_1 + (1-\varphi)a_2$ and $\beta = \varphi c_s + (1-\varphi)c_{ns} + 2s$. We take the first derivative of q_i^* with respect to n, $\frac{\partial q_i^*}{\partial n} =$

$-\frac{\beta n^2\gamma+2\alpha\beta n+\alpha\beta+2\alpha c}{n^2(\beta n+\beta+2c)^2}$. Because of the quadratic terms in denominator and all positive parameters, the first order condition $\frac{\partial q_i^*}{\partial n}$ is always negative, indicating the optimal ordering quantity for each manufacturer decreases with the number of the manufacturers. (ii): In terms of the total ordering quantity, we take the first derivative of Q with respect to n, $\frac{\partial Q}{\partial n} = -\frac{\beta(\alpha-\gamma)-2c\gamma}{(\beta n+\beta+2c)^2}$. Because of the quadratic term in denominator, positive parameters and $\alpha - \gamma < 0$, $\frac{\partial Q}{\partial n}$ is always non-negative. The total ordering quantity increases with the number of the manufacturers.

■