Task 879.1: Intelligent Demand Aggregation and Forecasting

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Outline

✓ Dynamic demand disaggregation
  • Fundamental study of demand planning approaches
Problem Description

1. Aggregating demand for better forecast
   - Total Forecast
   - How to disaggregate?

2. Effect of Product Life Cycle
   - Total Forecast
   - How to Consider PLC Effect in disaggregation?

Conventional Disaggregation Methods

- **Method-A**
  \[ P_{i,m} = \frac{\sum_{i=1}^{n} P_{i,t}}{n} \]
  (Average the Proportion of previous "n" periods to estimate the proportion next time period)

<table>
<thead>
<tr>
<th>Product</th>
<th>Time</th>
<th>Week 1</th>
<th>Week 2</th>
<th>Week 3</th>
<th>Total</th>
<th>Method-A</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td>10</td>
<td>20</td>
<td>30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>40</td>
<td>80</td>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>50</td>
<td>100</td>
<td>50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Proportion A</td>
<td></td>
<td>0.2</td>
<td>0.2</td>
<td>0.6</td>
<td>0.333</td>
<td></td>
</tr>
<tr>
<td>Proportion B</td>
<td></td>
<td>0.8</td>
<td>0.8</td>
<td>0.4</td>
<td>0.667</td>
<td></td>
</tr>
</tbody>
</table>

- **Method-B**
  \[ P_{i,m} = \frac{\sum_{i=1}^{n} d_{i,m}}{\sum_{i=1}^{n} D_{i,m}} \]
  (Average the demand of previous "n" periods to estimate the proportion next time period)

<table>
<thead>
<tr>
<th>Product</th>
<th>Time</th>
<th>Week 1</th>
<th>Week 2</th>
<th>Week 3</th>
<th>Total</th>
<th>Method-B</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>60</td>
<td>0.300</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>40</td>
<td>80</td>
<td>20</td>
<td>140</td>
<td>0.700</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>50</td>
<td>100</td>
<td>50</td>
<td>200</td>
<td></td>
</tr>
</tbody>
</table>
Proposed Methodology - EWMA

- Exponentially Weighted Moving Average statistic is introduced to catch the PLC

\[ w_t = \alpha(1 - \alpha)^{n-t} \]

- Exponential weights
- Parameter \( \alpha \) for exponential weight
- Time period \( t \)
- Number of total historical data \( n \)

- Different products have different \( \alpha \) values for best SSE performance.

Use of EWMA in Disaggregation

\[ \sum \text{Weights} = 1 \]
EWMA Disaggregation Formula

\[
\hat{P}_{t+1} = \frac{\sum_{j=1}^{m} \sum_{k=1}^{n} w_{j,k} \cdot d_{j,k}}{\sum_{j=1}^{m} \sum_{k=1}^{n} w_{j,k} \cdot d_{j,k}}
\]

\[
\sum_{j=1}^{m} w_{j,k} = \sum_{j=1}^{m} \alpha_i (1 - \alpha_i)^{n-1} \frac{1}{\sum_{j=1}^{m} \alpha_i (1 - \alpha_i)^{n-1}}
\]

Apply EWMA weights to historical "demand"
Sum of all EWMA weighted demands
Exponential weights

Case Study

1. Time horizon: 46-weeks semiconductor demand data.
2. Methods: conventional A, B; EWMA-A, EWMA-B
3. Historical data to determine proportions: 30 weeks data
Best Approach in Case Study: EWMA-B

The result shows that:

EWMA-B has the smallest MSE (best performance)

<table>
<thead>
<tr>
<th>MSE Comparison</th>
<th>Total MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method-B</td>
<td>4,407,671</td>
</tr>
<tr>
<td>Method-A</td>
<td>5,572,988</td>
</tr>
<tr>
<td>EWMA</td>
<td>1,567,397</td>
</tr>
</tbody>
</table>

**Question:** how to determine, dynamically if possible, the value of $\alpha$?

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**Determination of “$\alpha$” – PLC Indicator (Sample Auto-Correlation)**

$$SAC = \frac{Sample	ext{ Autocovariance}}{Sample	ext{ Variance}}$$

- SAC is the correlation between the two consecutive data in the same data series
- SAC $\nearrow$ when the data trend is significant
- SAC $\searrow$ when data is without a trend (stable)
Characteristics of Industrial Demands

1. Effect of PLC
2. “Standard deviation of demand is proportional to demand mean” (D. C. Heat & P. L. Jackson), (R. G. Brown)

Product demand at different time periods can be seen as different distributions with specific mean and standard deviation that is proportional to its mean.

3. Product Substitution within the product family

The Simulated DRAM Demand Dataset

- 3 products, 150-week demand data
- Product-1 is simulated as 256MB
- Product-2 is simulated as 128MB
- Product-3 is simulated as 512MB
- Each phase is simulated about 50 week length (1 year)
SAC of Simulated Dataset

Real Semiconductor Demand
Semiconductor Product Proportions

Performance metric: Proportion Mean Squared Error

\[ PMSE = \frac{1}{k} \sum_{i=1}^{n} \sum_{t=1}^{T} (\hat{P}_{i,t} - P_{i,t})^2 \]

- \( P_{i,t} \): Proportion of product "i" at time "t"
- \( \hat{P}_{i,t} \): Estimated proportion of product "i" at time "t"

Testing Results:

<table>
<thead>
<tr>
<th></th>
<th>Simulated Data</th>
<th>Real Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total PMSE</td>
<td></td>
</tr>
<tr>
<td>Conventional Method</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Method A</td>
<td>0.000146</td>
<td>0.000166</td>
</tr>
<tr>
<td>Method B</td>
<td>0.000666</td>
<td>0.001467</td>
</tr>
<tr>
<td>PIDE Method</td>
<td>0.001602</td>
<td>0.000161</td>
</tr>
</tbody>
</table>
Outline

• Dynamic demand disaggregation
✓ Fundamental study of demand planning approaches
Critical Statistical Properties

- Predictable Trend (PT): sum of autocorrelations over 30 lags
- Correlation (ρ)
- Coefficient of Variation (CV): degree of fluctuation

Predictable Trend (PT)

- Autocorrelated time series: PT
- Statistical Forecast Effectiveness
- Aggregated time series:
  - PT’s of individual demands are close
  - Forecast accuracy
Example

- Two AR(1) demands ($X_1$ and $X_2$).

  \[
  PT_{x_1} = PT_{x_2} = 4, \quad \rho = 0
  \]

  \[
  \begin{align*}
  \text{MSE of } X_1 & = 25.25 \\
  \text{MSE of } X_2 & = 24.62 \\
  \text{FSE} & = 9.99
  \end{align*}
  \]

Counter Example

- Two AR(1) demands ($W_1$ and $W_2$).

  \[
  PT_{w_1} = 4, \quad PT_{w_2} = -0.25, \quad \rho = 0
  \]

  \[
  \begin{align*}
  \text{MSE of } W_1 & = 51.35 \\
  \text{MSE of } W_2 & = 11.93 \\
  \text{FSE} & = 10.62
  \end{align*}
  \]
Demand Correlation $\rho$

- Correlation ($\rho$):
  \[
  \rho = \frac{\sigma_{x_1,x_2}(0)}{\sqrt{\sigma_{x_1}(0)\sigma_{x_2}(0)}}
  \]
  where $\sigma_{x_1,x_2}(0)$ is the covariance of demand series $X_{1t}$ and $X_{2t}$

- When $\rho$ is strong and positive, the predictable trend will be enhanced by aggregation and result in better forecast.

Example

- Two AR(1) demands ($M_{1t}$ and $M_{2t}$).
  \[
  PT_{m1} = PT_{m2} = 4, \rho = 0.92
  \]

MSE of $M_{1t}$ = 11.34  MSE of $M_{2t}$ = 12.91
FSE = 7.51
Coefficient of Variation: CV’s

- CV: measuring the degree of fluctuation
  \[ CV = \frac{\text{Standard deviation}}{\text{Mean}} \]

**Theorem 1:** CV inheritance after mean-proportional disaggregation

Let \( X_1_t \) and \( X_2_t \): two interrelated time series

\[ Y_t = X_1_t + X_2_t \]

By mean-proportional disaggregation:

\[ X'_1_t = \frac{\mu_1}{\mu_1 + \mu_2} \times Y_t \quad \text{and} \quad X'_2_t = \frac{\mu_2}{\mu_1 + \mu_2} \times Y_t \]

Then, \( CV_y = CV'_{x1} = CV'_{x2} \)

Individual CV’s Should be Close

- \( CV_{x1} = 0.097 < CV_{x1} = 0.508 \)
- \( CV_y = CV'_{x2} = 0.228 >> CV'_{x2} \)
- \( CV_y = CV'_{x2} = 0.028 << CV_{x1} \)

\( CV_{x1} \approx CV_{x2} \) is preferable

\( CV_{x1} = \frac{CV'_{x2}}{CV_{x1}} \approx 1 \) is preferable
The CV after Disaggregation Should be Smaller than the Original CV

- Forecast is to predict trend, not the noise.
  \[ x_t = 20 + 0.8 \cdot x_{t-1} + a_t \] where \( a_t \sim N(0,5^2) \)
  - The best forecast: \( \hat{x}_{r+1} = 20 + 0.8 \cdot x_r \) (Trend)

- The best forecast CV < Original CV
  \[ CV'_{r1} < CV'_{r2} \quad \& \quad CV'_{r1} = CV'_{r2} < CV'_{r2} \]
  \[ CV'_{r1} < 1 \quad \& \quad CV'_{r2} = CV'_{r2} < 1 \] are preferable

Evaluation Scenarios

- Demand Model:
  \[
  \begin{bmatrix}
  x_{1t} \\
  x_{2t}
  \end{bmatrix}
  =
  \begin{bmatrix}
  c_1 \\
  c_2
  \end{bmatrix}
  +
  \begin{bmatrix}
  \phi_{11} & \phi_{12} \\
  \phi_{21} & \phi_{22}
  \end{bmatrix}
  \begin{bmatrix}
  x_{1t-1} \\
  x_{2t-1}
  \end{bmatrix}
  +
  \begin{bmatrix}
  a_{1t} \\
  a_{2t}
  \end{bmatrix}
  \]

- 14 Scenarios for evaluation

  - Interrelated demands
  - Unilaterally related
  - Independent demands